

Conceptualizing and Assessing Curriculum Embedded Mathematics Knowledge

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Abstract

This 4-year collaborative project explores the question: What is required for teachers to use mathematics curriculum materials in ways that support and enhance mathematics instruction? We present preliminary findings from Year 1 seeking to identify the kinds of capacities (knowledge, abilities, ways of understanding, acting) necessary to use curriculum materials productively and fruitfully to enhance student learning. In particular, this poster centers on conceptualizing and measuring teacher knowledge required for interpreting tasks and representations in elementary mathematics curriculum materials. We also highlight challenges that we faced in this conceptual and developmental work.

Background

The project question emerges from contemporary trends to rely on published curriculum programs to guide teachers, improve instruction, and increase student learning. The expectation that curriculum materials can influence instruction is predicated on the assumption that teachers use them in ways that are true to the expectations of the curriculum designers. Research, however, has revealed substantial variation in how teachers use mathematics curriculum materials.

Brown (2002) proposed the term *Pedagogical Design Capacity* (PDC) to characterize an individual teacher's ability "to perceive and mobilize existing resources in order to craft instructional contexts." PDC includes both teacher knowledge (subject matter and pedagogical content knowledge) and the ability to act with and on that knowledge. Researchers have yet to articulate the dimensions of PDC for the purpose of measuring it, studying it, and developing it in teachers.

In Year 1, we developed a framework for and a tool to measure *Curriculum Embedded Mathematics Knowledge* - teachers' understanding of the mathematics underlying tasks, instructional designs, and representations in elementary mathematics curriculum materials. A central assumption of our work, and an ongoing principle in our decision making processes, was that reading mathematics curriculum resources to guide instruction calls on a kind of mathematical knowledge that involves identifying and grasping the mathematical meaning and potential of tasks that is not always made explicit. We think of this knowledge as a part piece of mathematics knowledge for teaching (Ball, Thames, & Phelps, 2008) that is highly situated in nature, and ultimately a component of PDC.

Curriculum Embedded Mathematics Assessment (CEMA)

- **CEMA:** a prototype of a new tool to measure teachers' understanding of mathematics embedded in curriculum resources (tasks, representations, teachers' guides, etc.).
- **Guiding questions:**
 - (1) How are mathematical ideas represented and embedded in various features of elementary curriculum programs?
 - (2) How are these ideas interpreted by elementary teachers?
- **Aim:** developing a *proof of concept* of the specialized knowledge needed to read the mathematics in curriculum materials and its relationship to Mathematics Knowledge for Teaching (Ball & Bass, 2003).
- **Structure:** excerpts and associated items - excerpts from five different elementary mathematics curriculum materials around which questions (items) about the mathematical intent and purpose are formulated (4-6 items per excerpt).
- **Five programs used:** (1) *Investigations in Number, Data, and Space*, (2) *Everyday Mathematics*, (3) *Trailblazers*, (4) *Scott Foresman Math*, and (5) *Singapore Mathematics*.
- **Content focus and grade level:** number and operations and algebra strands in grades 3-5.

Curriculum Embedded Mathematics Knowledge

We constructed a preliminary framework that would guide us in determining what questions to pose and how to formulate these questions. This framework went through multiple rounds of revision and was informed by knowledge of curriculum resources, how teachers engage with them (confirmed by teacher interviews), and previous literature (e.g., Ball, Thames, & Phelps; Grossman, 1991; Shulman, 1986; Sleep, 2009). This framework consists of four major dimensions:

1. **Mathematical Ideas** – Knowledge related to the mathematical ideas embedded in a particular task or student work; the ability to identify the mathematical point of a task or lesson.
2. **Surrounding knowledge** – Knowledge of how a particular mathematical goal is situated within a set of ideas, including the foundational ideas that undergird it and the future ideas that can be developed from it.
3. **Problem Complexity** – The ability to assess relative complexity and difficulty of a variety of mathematical ideas or tasks. The ability to categorize and order by increasing difficulty. The ability to identify possible points of confusion for learners associated with a given task.
4. **Connections Across Representations** – The ability to make connections across representations of the same mathematical idea, including narratives and symbolic representations.

Procedure

Project activities to develop the CEMA are summarized below. Even though these activities are provided in numerical order, the process was not completely linear. The results of later work led the project team to revisit earlier work in many cases.

1. Review and analyze 5 curriculum programs to identify key features.
2. Develop initial items using each of the 5 programs, focusing on its key features.
3. Develop a conceptual framework of the kind of knowledge to measure.
4. Construct 4 dimensions as components of the knowledge.
5. Interview teachers about curriculum reading to test and further refine the conceptual framework.
6. Identify excerpts from each curriculum and develop items for each excerpt.
7. Determine 8 excerpts with open-ended items for pilot.
8. Conduct pre-pilot with teachers and graduate students to ensure readability.
9. Revise the 8 excerpts and associated items based on the pre-pilot results.
10. Conduct the first pilot with 26 teachers (surveys).
11. Revise the excerpts and items, and create multiple-choices for each item based on the first pilot
12. Conduct the second pilot with additional 27 teachers (survey + interview)
13. Revise the excerpts and items based on the second pilot
14. Send the excerpts and items to math educators, mathematicians, and psychometricians for review
15. Revise the excerpts and items based on the external reviews
16. Conduct a field-test with 150 teachers on line (in Dec. 2010)
17. Analyze the items using teacher responses (in Jan. 2011)

Challenges

- Complexities of the work: conceptualization of the kind of teacher knowledge to measure and development of items that measure the kind of knowledge conceptualized
- Multiple rounds of refinement of the framework and the items
- Coordination of important issues related - mathematical precision, pedagogical use, and measurement purpose
- Creation of items based on common grounds (mathematical integrity), not depending on interpretation
- Determination of levels of mathematics: how much beyond the mathematics in grades 3-5