Teacher Fidelity Decisions and Their Impact on Lesson Enactment

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Abstract

This study examined *fidelity decisions* (FDs)—teachers' decisions on whether to use, modify, or omit each of the resources provided in the curriculum, or to add a new element to enact lessons—and their impact on lesson enactment within and across tasks and lessons. We particularly examined whether various FDs help teachers steer instruction to meet the mathematical goals of the lessons and whether they promote high cognitive demand. The findings of the study reveal teacher capacities that are needed to make appropriate FDs to transform the written to enacted lessons productively, which include recognizing important mathematical points and addressing them in instruction, and noticing and bridging gaps in the resources provided by the written lessons. Also, it is important for curriculum designers to make the goals and intentions of tasks, activities, and lessons as transparent as possible to teachers. Simply listing goals at the beginning of the lesson does not seem sufficient.

Teachers make various decisions when they use curriculum to plan and enact a lesson. First of all, they need to decide whether to use the task (lesson or unit) in the curriculum and, if so, how to use it. The curriculum usually includes various kinds of resources regarding how to enact the task (lesson or unit), such as questions to ask; representations, models, and strategies to use; a set of components of the task; and mathematical statements to make. Teachers decide whether to use, modify, or omit each of these elements provided in the curriculum. We call such decisions *fidelity decisions* (FDs), which indicate various possible adaptations teachers make as they use written lessons to design instruction. One important question to ask is how such FDs impact the quality of enacted lessons, or the quality of the transformation from the written to the enacted. Curriculum designers have specific intentions and mathematical goals to achieve through the written lessons. Certain FDs may be more critical to the quality of the enacted lessons.

In this study, we examined the kinds of FDs that particularly impact the enactment of the lesson, especially those that support or hinder the accomplishment of the goals of the written lesson and those that promote students' engagement at a high or low level of cognitive demand. Our research questions are: What fidelity decisions does a teacher make? What are the impacts of such fidelity decisions on the enactment of the written lesson?

Theoretical Foundation

The term *fidelity* indicates the alignment between the written and the enacted lessons in general (Remillard, 2005). Fidelity of curriculum implementation has been investigated from different perspectives, such as curricular coverage (Tarr, Chávez, Reys, & Reys, 2006), textbook integrity (Chval, Chávez, Reys, & Tarr, 2009), and fidelity to the authors' intended lesson and fidelity to the literal lesson (Brown, Pitvorec, Ditto, & Kelso, 2009). Unlike previous studies that focused more on overall implementation of curriculum, Brown et al.'s study examined whether critical elements of the lesson were implemented, in order to determine the level of fidelity in individual lessons observed.

It is important to analyze fidelity of curriculum implementation in small, meaningful chunks, such as tasks or lessons. We expand Brown et al.'s approach by investigating FDs teachers make at three levels (task, lesson, and unit) in order to see their impacts on the enactment of the written curriculum, especially how certain FDs support or hinder accomplishing the goals of the written lessons. A unit refers to a series of lessons on a particular topic, and each lesson is composed of tasks. By examining FDs at the task, lesson, and unit levels, we not only examine FDs and their impact on the enacted lessons in both zoom-in and zoom-out ways, but we also unpack the complexity of FDs in terms of meeting the mathematical goals in individual lessons as well as across lessons. Although we see the importance of Brown et al.'s examination of the authors' intended lesson, we focus on the goals presented in the written lesson. That is because teachers in general do not have easy access to the authors directly; rather, they have to rely on the written materials to interpret the goals of the lesson as presented in the curriculum.

Each written lesson has particular mathematical goals and objectives. Usually they are identified at the beginning of each lesson, which helps teachers "articulate the mathematical point" (Sleep, 2012) of the lesson. However, not all critical mathematical points are clearly identified or addressed in the lesson. Moreover, teachers with insufficient knowledge will have difficulty articulating the mathematical point of the lesson. Enacting the lesson toward the mathematical point is a challenging task for teachers.

In examining the impact of FDs on the enacted lessons, we also consider the cognitive demand of the enacted task. The level of cognitive demand indicates the kind of opportunity for students to learn (Stein, Grover, & Henningsen, 1996; Stein & Smith, 1998). Certain FDs increase, maintain, or reduce the cognitive demand of the task, which significantly influences the quality of the enacted lesson.

Besides goals and objectives, written lessons include various resources to support teachers to enact the lessons, such as directions and guidance to enact instructional activities, mathematical explanations, problems and tasks, representations, and strategies. We are particularly interested in three areas of resources: once they enact a task/activity in the written lesson, whether teachers (1) follow the directions and guidance to enact a task/activity; (2) offer mathematical statements and explanations; and (3) use models, representations, and strategies as they are suggested in the written lessons. Although models, representations, and strategies may be included in the directions and guidance, we separate them into a distinct group as they are important in addressing the mathematics in the written lessons and in the enacted lessons.

Teachers' use of curricular resources in individual lessons has been investigated (e.g., Brown & Edelson, 2003; Choppin, 2009, 2011; Lloyd, 2008; Remillard, 1999, 2000, 2005). A number of these studies focused on orientations teachers developed in using recommended resources when enacting lessons (e.g., Remillard & Bryans, 2004), and identifying types of adaptations teachers make (e.g., Forbes & Davis, 2010; Seago, 2007; Sherin & Drake, 2009). In our view, these different ways of curriculum use are indeed decisions teachers make whether to follow curriculum suggestions or introduce new elements of instructional design. We call them FDs, which include use, change, omission, and addition. *Use* occurs when teachers engage with curriculum suggestions almost as recommended; *change* occurs when teachers modify curriculum suggestions that significantly alter the intended meaning; *omission* occurs when teacher does not use curriculum suggestions; *addition* occurs when teachers make inputs not specified by the curriculum. We argue that teachers make these decisions because they think it will help them accomplish their goals for students. However, it is not known how such FDs affect teachers' orchestration of instruction to the mathematical point and opportunities for students to learn. Therefore, it is important to investigate the impact of FDs teachers make on the enacted lessons in terms of mathematical goals and cognitive demand of enacted tasks.

Methods

The data analyzed in this study were drawn from a project investigating teacher curriculum use, the *Improving Curriculum Use for Better Teaching* (ICUBiT) Project.

Teacher participants and curriculum programs. Data were gathered from 25 teachers in grades 3-5 using five different curriculum programs: (a) *Investigations in Number, Data, and Space*; (b) *Everyday Mathematics*; (c) *Math Trailblazers*; (d) Scott Foresman–Addison Wesley *Mathematics*; and (e) *Math in Focus*. The first three were NSF-funded programs; the fourth was commercially developed; the fifth was originally from Singapore and has gained popularity in the U.S. over recent years. The participant teachers had at least three years of teaching experience and at least two years of using the same curriculum program. This study drew on data from five teachers, one teacher per curriculum.

Data sources. The data we used in this study include classroom observations, teacher interviews (introductory and post-observation), and Curriculum Reading Logs (CRLs). Each teacher completed CRLs for each lesson that was observed: on a copy of the written lesson, the teacher indicated which parts they read as they planned instruction, which parts they planned to use, and which parts that influenced their planning. CRLs helped the researchers see plans for instruction and compare written and enacted lessons. Each teacher was observed for three consecutive lessons in each of two rounds. These enacted lessons were videotaped and transcribed. Also, each teacher was asked questions about his/her teaching experience and overall curriculum use at the beginning of the study, and then asked about specific teacher decisions in the observed lessons after each round of three observations. These interviews were audiotaped and transcribed.

Data analysis. The main part of the data analysis was coding teacher FDs and their impact on the enactment of the lesson. First, we chunked Written (W) and Enacted (E) tasks using CRLs and videotaped lessons, and created lesson analysis tables that included W- and E-tasks side-by-side. We defined a task as a chunk of activity (including teacher and student activity) aiming at an apparent distinct goal or product. In each pair of W- and E-tasks, we identified teacher FDs in the three main categories identified previously—whether each of them was used as recommended in the curriculum, changed, or omitted, or whether any new elements were added.

Once each task was analyzed regarding FDs, we examined how each FD influenced the enactment of the task, especially how each FD supported the accomplishment of the goals of the written lesson and affected cognitive demand (see Table 1). We also examined FDs in individual and multiple lessons (e.g., merging two tasks, changing the order of tasks, and omitting a lesson) and their impact on the goals of a series of lessons. Teacher interviews were analyzed to see teachers' intentions behind their decisions. After examining individual teachers, we searched for patterns in teacher FDs and their impact on lesson enactment.

Table 1
Codes for the Impact of FDs on Enacted Lesson

FDs' impact on	Code	Description
Lesson goals	1	teacher action toward goals
	0	teacher action not directly related to lesson goals
	–1	teacher action moving away from the mathematical point/goals
Cognitive demand	1	teacher action increasing the cognitive demand in the written lesson, or maintaining high level as in the written lesson
	0	teacher action not related to cognitive demand (neutral), or teacher action with no change of cognitive demand in the written lesson in case of low demand in the curriculum
	–1	teacher action decreasing cognitive demand in the written lesson

Results

We use one teacher's (Amy) case to illustrate kinds of FDs and their impact on the lesson enactment. Amy taught third grade using the second edition of *Investigations in Number, Data, and Space* (*Investigations*) (TERC, 2008). She had taught the curriculum for 6-7 years by the time she was observed. She was confident in using the curriculum and had an established practice of using it. In this section, first, we provide the overview of the written lessons and enacted lessons to explain FDs at the lesson and unit levels. Next, we describe specific FDs in enacting tasks and lessons along with their impacts on the mathematical goals and cognitive demand.

Overview of Written Lessons

The observed lessons were based on a series of six written lessons on an investigation (i.e., a series of lessons on a focused topic) of "Understanding Division" in the unit titled *Equal Groups: Multiplication and Division*. The previous three investigations in the unit are "Things That Come in Groups," "Skip Counting and 100 Charts," and "Arrays," in which students explore equal groups and multiplication. Table 2 provides the Math Focus Points, which is the term that the curriculum uses to highlight mathematical goals of the lesson, and instructional activities of the six lessons.

As shown in Table 2, there are a total of eight Math Focus Points (MFP) in the lessons. Overall, the written lessons emphasize the meaning of multiplication and division and the inverse relationship between the two operations in solving and creating multiplication and division story problems. All of the eight MFPs appear in more than one lesson, which means that MFPs are explored and developed through multiple lessons. It is important to understand why certain MFPs are repeated across lessons and how the MFPs are extended by building on previous lessons. For example, understanding the inverse relationship between multiplication and division and using it to solve problems require more than one lesson. Figure 1 presents the distribution of the MFPs across lessons. As seen in Figure 1, MFP 2 (i.e., using the inverse relationship between multiplication and division to solve problems) occurs most, which indicates that the lessons emphasize the relationship between the two operations as one of the most fundamental mathematical points students need to explore in these lessons. MFP 3 (i.e., using multiplication combinations to solve division problems), which is also emphasized, can be understood in relation to MFP 2. Therefore, it is clear that in the investigation (i.e., the six lessons) students are expected to understand division in relation to multiplication.

MFPs	Lessons 4.1	4.2	4.3	4.4	4.5	4.6
1	●		●			
2	●	●		●	●	●
3		●		●	●	●
4		●	●	●	●	●
5			●			
6			●			
7			●	●	●	●
8					●	●

Figure 1. Math Focus Points across lessons

Table 2
Sequence of the Lessons (*Investigations 4. Understanding Division*)

Lesson	Math Focus Points (MFP)	Lesson activities
4.1 Solving Division Problems	• Understanding division as the splitting of a quantity into equal groups (1)* • Using the inverse relationship between multiplication and division to solve problems (2)	1. Solving division story problems 2. Sharing our solutions
4.2 Multiply or Divide?	• Using the inverse relationship between multiplication and division to solve problems (2) • Using multiplication combinations to solve division problems (3) • Using and understanding division notation (4)	1. Solving story problems 2. Multiply or divide?
4.3 Writing Story Problems	• Understanding division as the splitting of a quantity into equal groups (1) • Writing and solving multiplication problems in context (5) • Writing and solving division problems in context (6) • Using and understanding multiplication notation (7) • Using and understanding division notation (4)	1. Introducing the class multiplication /division book 2. Writing problems for the class book
4.4 Missing Factors	• Using multiplication combinations to solve division problems (3) • Using the inverse relationship between multiplication and division to solve problems (2) • Using and understanding multiplication notation (7) • Using and understanding division notation (4)	1. Introducing missing factors 2. Playing missing factors
4.5 Solving Multiplication and Division Problems	• Using multiplication combinations to solve division problems (3) • Using the inverse relationship between multiplication and division to solve problems (2) • Using and understanding multiplication notation (7) • Using and understanding division notation (4) • Identifying and learning multiplication combinations not yet known (8)	1. Different ways to write problems 2. Practicing multiplication and division (workshop)
4.6 Solving Multiplication and Division Problems, continued	• Using multiplication combinations to solve division problems (3) • Using the inverse relationship between multiplication and division to solve problems (2) • Using and understanding multiplication notation (7) • Using and understanding division notation (4) • Identifying and learning multiplication combinations not yet known (8)	1. Workshop (continued) 2. Solving division problems

* The number in the parentheses indicates the same MFP across lessons.

Overview of Enacted Lessons

Figure 2 summarizes the three lessons observed in terms of activities used from the written lessons. Note that activities underlined and in bold are those omitted (e.g., **<u>A1</u>**). The teacher omitted Lesson 4.4 entirely. She used Lesson 4.5–Activity 2 and Lesson 4.6–Activity 1 partially, both of which were combined into one single activity in the third observed lesson. Those two activities include subcomponents that provide opportunities for practice, indicated as a, b, c, and d in Figure 2. According to the written lessons, students are expected to practice solving multiplication and division problems in each activity with the two separate incidences. By combining the two into one, the teacher provided fewer opportunities for practice. The teacher saw it as repetition and included it only once. She also omitted the other components of the two activities (i.e., Lesson 4.5–Activity 2b and Lesson 4.6–Activity A1a, c, d), which further limited diverse opportunities for practice. She also replaced Lesson 4.3–Activity 1 with an activity of generating key words (a1). Since students had difficulty with Lesson 4.3–Activity 2 (creating multiplication and division story problems), the teacher had to spend a large chunk of time on this again in the third lesson observed. She also omitted Lesson 4.5–Activity 1 and Lesson 4.6–Activity 2. Overall, she transformed six written lessons into four enacted lessons (including the one enacted before the first observed lesson) and, as a result, many activities were omitted.

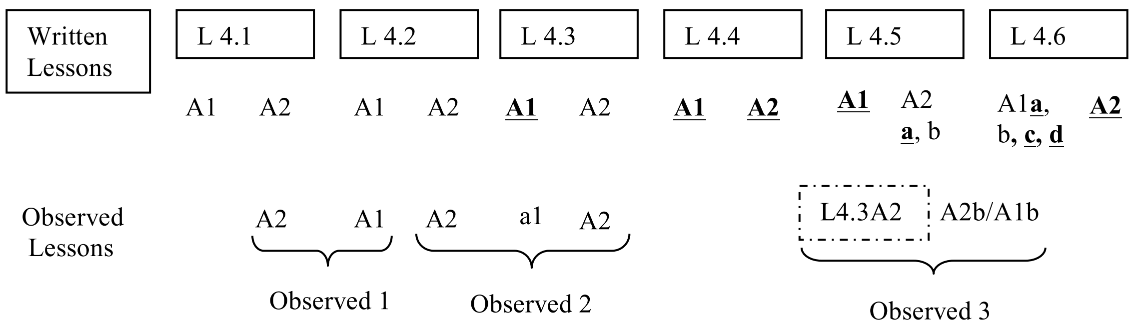

Figure 2. Written and enacted lesson activities

In fact, there were serious gaps between the lessons observed (activities used) and the written lessons. Overall, the teacher used activities with significant change in mathematical points of the lessons and activities. Also, she reduced the cognitive demand of the tasks/activities and the lessons. For example, in Lesson 4.1–Activity 2, which requires students to share various solutions for one of the problems they solved and reflect on the attributes of division problems, the teacher focuses on solutions to individual problems (one solution per problem), talking about all the six problems students solved as a way of checking students' solutions to the problems. She did not highlight the mathematical points (MFPs) and meaning across problems. That is, she did not bring up the relationship between multiplication and division, and did not emphasize the meaning of the two operations.

Fidelity Decisions and Their Impact

Table 3 summarizes fidelity decisions on the curriculum guidance (i.e., directions, mathematical statements, and models/strategies) and their impact on lesson enactment in terms of meeting the mathematical goals of the lesson (MFPs) and the cognitive demand that students are expected to engage in. Note that tasks/activities omitted from the written lessons were not used for coding in order to determine any omissions in the table since the entire task/activity was already omitted.

As shown in Table 3, most guidance/directions Amy added negatively affected the lessons enacted. Only 23 out of 133 additions of guidance coded (17.6%) were positive in both meeting the lesson goals and maintaining high cognitive demand, whereas 90 out of 133 (67.7%) were negative in both. Also, directions she changed and statements she omitted, added, or changed had negative impacts on the enacted lessons overall. In contrast, most of directions and models/strategies she used from the curriculum positively affected the lessons. This implies that in general using the resources in the written lessons helped Amy steer the instruction toward mathematical points and promote a high level of cognitive demand. Overall, change or omission of the guidance in the written lessons and addition of resources outside the curriculum (i.e., the written lessons) influenced the lesson enactment negatively. Below, we describe in detail the kinds of Amy's FDs and their impacts on her enacted lessons in the cases of omission, addition, change, and use, respectively. Although we explain them individually, the FDs Amy made in these different ways are closely interrelated, affecting the quality of the enacted lessons.

Table 3
Fidelity Decisions and Their Impact

Resources	Fidelity	Lesson Goals	Cognitive Demand	Frequency
Directions	Use	0	0	1
Directions	Use	1	1	27
Directions	Use	1	–1	4
Directions	Use	–1	–1	7
Directions	Change	–1	–1	4
Directions	Omission	–1	–1	13
Directions	Addition	0	0	4
Directions	Addition	1	0	1
Directions	Addition	1	1	23
Directions	Addition	1	–1	13
Directions	Addition	–1	1	1
Directions	Addition	–1	–1	90
Directions	Addition	0	–1	1
Statements	Use	1	1	3
Statements	Use	1	–1	1
Statements	Use	–1	–1	2
Statements	Change	0	0	1
Statements	Omission	–1	–1	3
Statements	Change	–1	–1	7
Statements	Addition	0	0	1
Statements	Addition	1	0	1
Statements	Addition	1	1	4
Statements	Addition	1	–1	3
Statements	Addition	–1	–1	13
Models	Use	1	1	10
Models	Use	–1	–1	1
Models	Omission	–1	–1	5
Models	Addition	0	0	1

Omission
In Amy's case, the most fundamental FDs that impacted the enacted lessons negatively were omitting lessons, activities, and curricular guidance. By doing this, she steered the lessons away from their mathematical points and guided students to just solve problems. Through problem solving and discussing their solutions, students should develop the desired understanding of the mathematical points identified by the curriculum designers. However, Amy focused on solving each problem and did not highlight important MFPs, such as using the inverse relationship between multiplication and division to solve problems.

The critical omissions she made include one lesson on arrays (Lesson 4.4), which helps students relate multiplication and division using the product and their factors. She also omitted a task in Lesson 4.2, which provides an introduction to writing multiplication and division story problems. The task especially brings up two related expressions (6 × 3 and 18 ÷ 3) and asks students in pairs to come up with a story problem for each. The focus is on the difference between the two operations and the task gives an opportunity to assess student thinking before assigning the task of creating multiplication and division story problems. The guidance explicitly indicates that teachers need to:

> Listen for student understanding of the difference between multiplication and division. For example, do the problems students make for the expression 18 ÷ 3 begin with the quantity 18 and divide it into 3 equal groups or groups of 3? Do the problems for 6 × 3 involve 6 groups of 3 or 3 groups 6? (p. 126)

Instead, she spent time on generating key words for multiplication and division. She made comments as students offered some expressions as key words, whether each suggestion would be acceptable in each operation. In doing so, she lost an opportunity to highlight characteristics of multiplication and division in relation to each other.

The loss of meaning continued as she enacted Lesson 4.2–Activity 2 (creating multiplication and division story problems). See the guidance in the curriculum below on how to intervene when students have difficulty generating their own multiplication and division story problems (MFPs 5 and 6).

> Help students talk through the elements of a multiplication situation (two known factors and an unknown product) and a division situation (product and one known factor). Write multiplication and division equations with small numbers and ask students to model the action of each with cubes. (p. 127)

This guidance is followed by the specific script shown below, to use during intervention.

> Look at this equation, 3 × 4=__ (or 12 ÷ 4=__). Can you show me with cubes what this problem would look like? Can you think of a situation to write about in which you might have 3 groups of 4 things (or 12 things divided into groups of 4 or 4 groups)? How can the class poster of "Things That Come in Groups" help you? (p. 128)

In the guidance above, it is clear that the two operations deal with equal groups and the product and that the two operations have an inverse relationship between them. However, in her intervention, omitting the entire guidance, Amy did not highlight the critical aspect of the operations. Rather, she focused on using key words to determine which operation a given problem required or to create multiplication and division story problems.

Other critical omissions are mathematical statements, directions (teacher questions), and models. In explaining division, the curriculum highlights that "The answer is the number of groups or the number of items in each group" (p. 119). Also, relating multiplication and division, a chart is used with specific terms such as *number of groups, number in each group, product,* and *equation*. According to the guidance, the teacher is supposed to help students to "recognize that the unknown information for this problem is the product (the number of yogurt cups in all)" (p. 124). In fact, the teacher rarely used such expressions in explaining multiplication and division, and, as a result, many students were not clear about what makes an operation multiplication or division. For example, when creating a story problem for multiplication, several students did not understand that they had to use equal groups. A story problem like, "I have 10 apples and my friend has 5 apples. How many do we have in all?" indicates that students did not know how multiplication problems are different from addition problems. Relying on key words, to the students "how many in all" could be sufficient to make a multiplication story problem.

While focusing on and creating key words for solving problems, the meanings of the two operations were only implicitly shared. When much confusion was apparent among students while generating multiplication and division story problems, Amy intervened with many struggling students often focusing on key words. She did not explicitly mention that multiplication and division deal with equal groups, using expressions such as *equal groups, number in each group,* or *product*. At best, she said:

> If you have 10 and he has 5 and we want to know how many in all, we're just putting them together. So that's just adding. But if you have a pack of 10 and a pack of 10 and he has a pack of 10 and a pack of 10 then you've got 10, 10, 10, 10. Which is multiplying. Does that make sense, sweetheart?
> ….
> So, multiplication means I have something that has a certain number of somethings in it. Like, I have three packs of gum, each pack has 5 pieces. (Second observed lesson)

She also never brought up how a known multiplication combination could be used to solve a division problem (MFP 3). This helps students see and use the inverse relationship between the two operations in solving and generating multiplication and division story problems. Throughout the lessons, she failed to highlight this important mathematical idea. She also failed to recognize this idea even when students brought it up. Solving a multiplication problem of 5 × 7 that used the same combination of numbers in a previous division problem, a student responded that 5 × 7 = 35 by using the related division problem they solved. The teacher began the problem without using this relationship, as if this was a totally different problem. She repeatedly asked, "How do you know that?" Then, they counted by 5s again exactly the same way they did in the previous problem for 35 ÷ 5 = 7.

Teacher: … So how do I solve this problem? Scott? …
Student: Multiply 5 times 7.
Teacher: 5 times 7 and that's gonna equal something but we don't know what yet. Okay?
Student: 5 times 7 equals 35.
Teacher: 5 times 7 equals 35? How do you know?
Student: 35 divided 7 equals 5 [35 divided by 5 equals 7].
Teacher: But how do you know that?
Student: Count by 5's.
Teacher: Go ahead. So go ahead. 5…
Student: 5, 10, 15, 20, 25, 30, 35.

Also, she did not ask critical questions, such as "Describe this problem. What information do you know about this problem? What do you need to find out?" Instead, she asked, "What is our key word? Is this multiplication or division?"

The teacher did not push for multiple strategies at least during the whole group discussion. This is a serious neglect of the curriculum's pedagogical approach. She did not provide students with an opportunity to share multiple strategies and compare them. Her students were not offered a critical strategy of using a multiplication combination to solve a division problem. She talked about all the problems students were asked to solved, one solution per problem based on students' response, rather than focusing on one or two with multiple strategies as suggested in the curriculum.

Addition
Some guidance, questions, and statements Amy added were effective. They promoted students' understanding and required high cognitive demand. For example, every time when solving a story problem, Amy asked students to visualize the problem situation by closing their eyes and imagining what is happening in the problem context.

> To share equally because, here again, get that picture in your head. Kind of close for a second, imagine you and your two best friends standing on either side of you. Mom gives you a deck of 18 playing cards and you're gonna pass them out. … That's exactly what's gonna happen, right? I'm seeing me and my two best friends and Mom's standing in front of me and she's going, "Ok, here's one for you, one for you, one for you." We're gonna share them equally. So we we're taking those cards and dividing them up among our friends. Do you agree? … Do you see it now? (first lesson observed)

The written lessons include specific guidance, such as "encourage students to act out the action of each problem, using cubes or drawings" (p. 122), to help students understand what the problem is asking them to do. However, imagining the problem situation to figure out what they need to do to solve the problem was her own addition based on her colleague's suggestion at the school district meeting. This visualization helped students see what is happening in the problem, encouraging students to think about the meaning embedded in the problem and relating that to an operation.

She also asked some critical questions that were not included in the written lessons. These questions prompted students to relate the solution process with the problem. See the following excerpt.

> Teacher: Okay, 35 divided by 5. Okay, so how do I solve it?
> Student: Um, you count by 5's until you hit 35.
> Teacher: Okay, go ahead.
> Student: 5, 10, 15, 20…
> Teacher: Slower, slower, slower. You can talk faster than I can write. Okay, go.
> Student: 25, 30, 35.
> Teacher: Okay so *35's our answer, right?* Yeah? So 35 divided by 5 is 35.
> Student: No, that's not right.
> Teacher: Oh, okay.
> Student: The answer is 7.
> Teacher: *How do you know that*?
> Student: Um, because you count by 5's and how many 5 goes into 35.
> Teacher: See it? 1, 2, 3, 4, 5, 6, 7. It's how many times you counted by 5. So, our answer is 7.

Students counted by 5s and reached the target number, 35, as a way to solve 35 ÷ 5. Instead of determining the answer right away, the teacher asked students now what the answer was to the given problem and how they knew that was the answer. She provided students an opportunity to step back from counting by 5s and relate that to the given problem. She also added a statement, "1, 2, 3, 4, 5, 6, 7. It's how many times you counted by 5. So, our answer is 7." This highlighted the mathematics embedded in the skip counting, i.e., how many groups of 5 are in 35 tells the answer to 35 ÷ 5.

However, as mentioned previously, her addition of using key words in the lessons significantly minimized the positive impact of her added guidance, questions, and statements. Emphasizing key words throughout the lessons, Amy replaced activities and directions with those around key words. For example, she replaced an activity of generating and discussing story problems for 6 × 3 and 18 ÷ 3 with generating words and expressions that cue multiplication or division. She asked students to underline key words and determine which operation to use, and find "the numbers" in the problem to execute the operation determined. She also made problematic statements usually around key words, such as, "If it says 'in each,' it's gonna be a division problem." She made students' problem solving mechanical in this way—find key words, determine the operation to use, find the numbers to use, solve the problem, and write the number sentence, reducing cognitive demand greatly, and misguiding students' thinking about multiplication and division.

Emphasizing key words, sometimes Amy herself was confused while explaining the distinction among different key words and brought up other operations to consider, such as addition, which created more confusion among students. Despite that, she continued to emphasize key words when intervening struggling students: “Now remind me, what are our multiplication key words? If it’s a multiplication story problem it’s gonna have what key words in it?”

As a result, after spending two days generating multiplication and division story problems, still more than half of her students were not able to complete the task. On the third day of classroom observation, there was a range of student-generated story problems. Some students had stories but no questions; some students did not have multiplication or division contexts (addition or subtraction instead); some students had numbers that do not work well (34 things divided equally into 3 or 4 groups); students had only one type of story problem (both multiplication or both division).

Change

Her significant change of given resources was mostly around mathematical goals of the lessons and mathematical statements provided in the written lessons. She used the problems and tasks provided in the written lessons, but the way she used them altered MFPs significantly. She also changed goals of discussion and moved away from the MFP that should be highlighted through discussion. She stated that at the beginning: “The reason being the primary objective of us correcting these papers is so that you can talk about what the key words are when you’re creating a multiplication or division problem.” In fact, the written lesson directs teachers to have students share their solutions to two particular related problems (one multiplication, $4 \times 5 = 20$, and one division, $20 \div 4 = 5$) and highlight what each operation means and how they are related to each other. Instead, Amy went through all the problems students solved, one by one, to identify key words and determine which operation to use.

Also, she omitted most of the important mathematical statements or changed them to promote a different meaning. In particular, Amy significantly altered mathematical statements provided in the written lessons when she highlighted the mathematics students need to learn or use in the lessons. For example, she mentioned several times, “Division sentence always starts with the biggest number.” The statements included in the written lessons are: “Each division problem gives a total that must be divided into equal groups. The answer is the number of groups or the number of items in each group” (p. 119). Certainly Amy altered the meaning of division that students need to learn and did not highlight the important attribute of division—dealing with equal groups.

Overall, her changed directions to guide students to engage in the mathematics of the lessons and altered mathematical statements greatly minimized students’ learning opportunity in the lessons of multiplication and division in the way they were designed.

Use

Amy used problems, tasks, and activities provided in the written lessons. She assigned them to students and discussed the meaning of each problem by helping them visualize the problem situation and solutions to the problems. She also had MFPs in her mind as she read them while preparing for the lessons. She also used models, representations, and strategies that were included in the written lessons, such as drawing pictures of equal groups and counting by a certain number to determine the product or the number of groups. However, her use of resources (lessons, tasks, directions, mathematical statements, models) was mainly based on decisions on what to do, not necessarily about how to do. For example, she did not follow the guidance regarding how to use the problems to highlight the meaning of multiplication and division and the inverse relationship between the two operations. Most of directions and guidance were ignored when they addressed mathematical points embedded in the problems and how to help students understand the big ideas and complete their tasks using the big ideas. Although Amy read the guidance, she did not clearly relate directions with identified MFPs. Likewise, she rarely used mathematical statements that highlight the relationships and meaning of multiplication and division. Instead, she omitted or altered those important statements and often added inaccurate statements that are not included in the written lessons (e.g., “division starts with the biggest number”).

To summarize, basically Amy altered the written lessons on multiplication and division significantly and did not meet the many of MFPs sufficiently. Although at times she made appropriate adaptions that supported mathematical goals and high cognitive demand, her ignorance or alterations of critical resources, such as directions and mathematical statements, as well as inappropriate additions caused her to fail to create opportunities for students to learn the meaning of and relationship between multiplication and division.

Discussion

Teachers can make various FDs depending on their classroom situation. However, such decisions need to be made in accordance with the mathematical goals of the lesson. The case of Amy highlights that making appropriate FDs (use, omission, change, and addition) greatly depends on teacher capacity of recognizing important mathematical points and addressing them in instruction. This result is aligned with what Sleep (2012) refers to “mathematical purposing.” According to her, mathematical purposing involves articulating the mathematical point and designing instruction to the mathematical point. Essentially, making right FDs is based on understanding the mathematical goals of the lesson and determining which guidance to use and how to use it in order to teach the lesson toward the goals.

The results of the study also reveal that making appropriate additions to enact lessons requires teacher capacity of noticing and bridging gaps in the guidance provided by the written lessons. Amy, as described in this study, made numerous additions to the written lessons while enacting them. Some of them positively influenced the enacted lessons; others hindered meeting the lesson goals and reduced the cognitive demand of the task. For example, she asked students what their answer would be once they skip counted by 5s as a way to solve 35 ÷ 5. The written lessons include skip counting as a strategy and provide examples of using this strategy. However, there is no additional guidance regarding how to talk about this strategy with students. Amy specifically added an important question that made students reflect on the skip counting and how that leads to the solution to the given problem. In contrast, she thought that it was important to add key words that were not specified in the written lessons. She determined to add this element to the lessons to help students know which operation to use. This decision indicates a lack of her understanding of the MFPs, which influenced her to misidentify the gap.

As evident in the MFPs of the lessons Amy enacted, repeated goals across lessons are for building a mathematical storyline. Using the inverse relationship between multiplication and division to solve problems (MFP 2) extends across five of the six lessons. In the very first written lesson, MFP 2 is introduced through a strategy of using a known multiplication combination to solve a division story problem. In the second lesson, MFP 2 is more explicitly addressed in an instructional activity of discussing how multiplication and division are related to each other by using two related multiplication and division problems ($4 \times 5 = 20$ and $20 \div 4 = 5$) students already solved. Although the third lesson does not include MFP 2 as one of its MFPs, the lesson guides teachers to encourage students to think about the relationship between multiplication and division and the meaning of the two operations in relation to each other before the activity of generating multiplication and division story problems. The subsequent three lessons indicate MFP 2 as their mathematical goal and students are asked to use the relationship between the two operations to complete tasks and activities. Seeing how MFP 2 is addressed across the lessons is important in developing a storyline regarding this mathematical point. When teachers do not see this mathematical connection across the lessons, it is not likely that they build a well-articulated mathematical storyline through enacted lessons. Teachers need to understand the goals of each lesson in relation to building a mathematical storyline across lessons.

In fact, capacities needed to make good FDs that we described above elaborates Brown (2009)'s notion of Pedagogical Design Capacity (PDC), which he defines as "a teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" (p. 29). He further describes that "PDC describes the manner and degree to which teachers create deliberate, productive designs that help accomplish their instructional goals" (p. 29). Therefore, examining kinds of FDs and their impacts on instruction and providing appropriate teacher preparation and education will support teachers to develop PDC that is needed for productive curriculum use.

Curriculum designers need to make the mathematical point of the lesson clear in terms of when it is introduced and how it is developed through a series of lessons. Although MFP 2 is greatly emphasized in the written lessons that Amy used to teach multiplication and division, it is not clear how MFP 2 is met in Lesson 4.1. In this lesson, a strategy is included of using a known multiplication combination ($4 \times 5 = 20$) to solve a division problem ($28 \div 4$), that is, creating 5 groups of 4 and then adding some more groups of 4 to find the answer. Other than including this strategy, this lesson does not clearly indicate why MFP 2 becomes an important mathematical goal to accomplish and how this goal can be met. Subsequent lessons do not provide clear explanations either, although instructional activities in the subsequent lessons more explicitly target this MFP. Without sufficient knowledge, teachers may miss this important mathematical point in instruction, as Amy did, and teachers will greatly benefit from explicit explanations about mathematical points and how lesson activities accomplish them in the written lessons.

The findings of the study suggest that professional development on curriculum use is needed even for teachers who have used the given curriculum for a long time. Amy recognized the usefulness of visualization when she heard that at the district's meeting and used it in her teaching. She was open to suggestions and tried to learn new things, but she still had past habits of and beliefs on accustomed teaching moves and decisions. She thought that key words really helped students understand what multiplication and division are, although she noticed her students struggled a lot. Teachers like Amy need revisited professional development highlighting the essence of the teaching approaches and rationale behind each activity and connections across activities and lessons. For the first couple of years using a new curriculum, teachers read carefully; later they tend to rely on their past experience and colleagues, rather than using curriculum carefully. Missing important elements in the lessons, Amy confessed that she just skimmed through the lessons, as she had already taught them for 6-7 years. This justifies that professional development on using a curriculum is necessary for experienced users as well.

This study explored what kinds of FDs teachers make and how they impact lesson enactment within and across tasks and lessons. We particularly examined whether various FDs help teachers meet the mathematical goals of the lessons and whether they promote high cognitive demand. The findings of the study reveal particular teacher capacities that are needed to make appropriate FDs to transform the written to enacted lessons productively. Also, it is important for curriculum designers to make the goals and intentions of tasks, activities, and lessons as transparent as possible to teachers. Simply listing goals at the beginning of the lesson does not seem sufficient.

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