A Comparative Analysis of Mathematical and Pedagogical Components of Five Elementary Mathematics Curricula

Improving Curriculum Use for Better Teaching Project (ICUBiT)*

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This paper presents findings from a preliminary analysis of five elementary mathematics teacher's guides from the perspective of both the demands placed on the teacher and the supports they provide the teacher. The ICUBiT study (Improving Curriculum Use for Better Teaching) seeks to conceptualize and study pedagogical design capacity—a teacher's ability to effectively utilize existing curricular resources to design instruction (Brown, 2009). Because we understand teachers' curriculum use to be a participatory process in which teachers and curriculum resources interact, a teacher's pedagogical design capacity must be dependent, to some extent, on the particular curriculum resource being used (Remillard, 2005). As designed instructional guides, curriculum materials provide teachers tools to guide instruction. Making sense of and using these tools to design and enact instruction places a demand on teacher capacity. Stein and colleagues examined two elementary programs and noted that different curriculum programs varied in the demands they placed on teachers who used them (Stein & Kim, 2009). They also found that curriculum materials varied in the amount and kind of support they provided teachers to meet these demands. More recently, Stein and Kaufman (2010) found that teachers varied in the way they used these supports; and these variations significantly affected the quality of instruction.

This study builds on Stein & Kim's (2009) analysis of two *Standards*-based curriculum programs, analyzing five elementary programs, three of which can be classified as Standards-based (Senk & Thompson, 2003), one of which is commercially developed and published, and the last, Singapore Math, was adapted from the materials used in Singapore and marketed to an U.S. audience. The questions guiding our analysis were:

1. What does the curriculum reveal about the "author-intended" curriculum? (Brown, et al., 2008)
2. What demands does the author-intended curriculum place on teachers as users and enactors of the curriculum?
3. What supports does the curriculum provide the teacher to aid in enacting the intended curriculum?

These questions and our analytical framework grow out of the existing research on curriculum materials as tools to support teacher learning and pedagogical reform. Textbooks and curriculum materials have long been seen as tools of pedagogical reform, in part because they are so widely used in U.S. classrooms, especially in mathematics. The most recent national reform effort, in which new curriculum materials received renewed attention in the form of federal investment in material development, was the Standards-based reforms, prompted by the national embrace of the NCTM Standards (NCTM, 1989, 2000). The National Science Foundation invested extensively in several curriculum design initiatives under the Instructional Materials Development Program to support the development of new materials aimed at supporting teachers in schools adopting content and pedagogical practices promoted by the new Standards.

As these new programs found their way to the market and schools across the U.S. in the mid 1990's, they generated enthusiasm, attracted critics, and prompted a variety of questions about their effects on student learning (Stein, Remillard, & Smith, 2007). Some studies have examined the effects of these and other programs on student learning (e.g., Riordan & Noyce, 2001; What Works Clearinghouse, 2007).

Studies such as these have been criticized for not taking into account the extent to which the particular programs were implemented as intended (NRC, 2004). In a review of research on the achievement effects of any curriculum programs, an NRC report found insufficient information to determine the effectiveness of any program and called for studies to include a measure of integrity of implementation. Indeed, this call appears warranted given the variation in implementation found by many studies of teachers using standards-based curriculum materials (e.g., Collopy, 2003; Lloyd, 1999; Remillard & Bryans, 2004; Stein, Remillard, & Smith). One study that does take into account curriculum integrity (Tarr, et al., 2008) treats standards-based curriculum materials as one type, contrasting them to non-Standards-based programs. Thus far, very few studies have considered the role that a variety of mediating factors, including differences across standards-based materials themselves, play in different levels of use of curriculum resources.

Stein and Kaufman's (2010) recent study attempts to do so by comparing the quality of curriculum implementation of two standards-based programs, *Investigations in Numbers, Data, and Space* (INV) and *Everyday Mathematics* (EM). These two programs make for a revealing comparison because they are designed quite differently with respect to the cognitive demand of the mathematics tasks and the extent to which the authors provide support for the teacher in using these tasks. Davis and Krajcik, (2005) refer to these features as "educative" for the teacher, a term based on Ball and Cohen's (1996) call for curriculum designers to consider ways to support teachers in the process of enacting curriculum. (See Stein & Kim, 2009, for more details on differences between the two programs.) Stein & Kaufman found that when teachers attended to the educative features in the Investigations curriculum, particularly the elaborations of the central mathematical ideas, a feature more prominent in Investigations than Everyday Mathematics, the quality of implementation was significantly higher.

Analytical Framework

In conceptualizing the demands that curriculum resources place on the teacher, as well as the potential forms of support, we identified two key elements of the guidance provided: a) the model lesson and b) the voice of the text. Our view of the model lesson is related to what Brown, et al. (2008) refer to as the "researchers' model of the author-intended lesson." They use this term to describe a model of the curriculum, at the lesson level, as it might have been imagined by the designers of the curriculum. This model is clearly subject to interpretation and the extent to which it can be easily discerned is dependent on the degree of specificity in the text. Brown and her colleagues offer a framework for how researchers develop models of lesson intentions that begins with the philosophical statements made in the curriculum as seen through the various lenses brought by the researcher to create the "researcher's image of the curriculum's philosophy." This general image of the curriculum philosophy can then be applied to examining written lesson materials to produce "the authors' intended curriculum at the lesson level" (p. 370).

We do not think of the author-intended lesson (what we are calling the model lesson) as a narrative description of a lesson, but as being comprised of a set key dimensions or elements that are critical for capturing the mathematical and pedagogical components of the lesson and, when taken together across multiple lessons, the curriculum. The extent to which these dimensions are present for a given curriculum might be used to determine the quality of the use of the curriculum. In our analysis of the model lesson, we focus on four dimensions: a) the mathematical emphasis or focus; b) the cognitive demand of the central tasks of the lesson, c) the primary instructional representations used to communicate mathematical ideas, and d) the instructional approach, which includes the teacher's and students' roles throughout the lesson, the role of the text, and expectations for how learning takes place.

The mathematical emphasis refers to the aspects of mathematics knowledge and practice that are valued in the curriculum. The cognitive demand of the tasks (Stein, Grover, & Henningsen, 1996) provides insight into the mathematical and related pedagogical complexity of the tasks. High demand tasks are intellectually and conceptually challenging and place emphasis on underlying concepts, patterns, and properties. High demand tasks can be classified as *Doing Mathematics,* which involve nonroutine thinking, or *Procedures with Connections,* which emphasize underlying meaning within procedural routines or practices. Low demand tasks can be classified as *Procedures without Connections* or *Memorization*, both of which focus on routine and procedural elements of mathematical tasks, often in isolation, and without connections to mathematical sense making. (See Stein, Smith, Henningsen, & Silver, 2000, for more details.). The instructional representations are the models (visual, concrete, pictorial, symbolic, narrative) used to represent the mathematical ideas being taught. The instructional approach refers to the implicit and explicit ideas about how and where learning occurs and the roles that the teacher, the students, and the text play in this process. Our thinking about this final dimension is informed by Stein &Kim's (2009) analysis of EM and INV. As a result of their analysis, they argue that EM is designed such that the learning takes place primarily between the student and the curriculum while the teacher plays a supporting role; whereas, INV is designed so that the learning takes place in the interactions between the students and the teacher, in which the text plays a supporting role. The set of dimensions that comprise the model curriculum determine the mathematical and pedagogical demands the curriculum places on the teacher.

The voice of the text refers to how the text communicates to the teacher, what it communicates about, and how the text positions the teacher as a reader and user of it (Herbel-Eisenmann, 2000; Love & Pimm, 1996; Remillard, in press). Typically, the teacher's guide of a curriculum program provides the teacher with guidance in enacting the lesson. Remillard (2000) refers to this as *talking through* the teacher or communicating with the teacher through guiding her actions. Curriculum designers seeking to provide additional supports for teachers might also include information that *talks to* the teacher about design decisions or important ideas. Davis and Krajcik (2005) offer several ways that authors can speak *to* teachers that we have grouped in the following categories: a) make the developers' design decisions visible, b) help the teacher anticipate and interpret what learners may think or do, c) support teachers in learning more about the content, or d) support the teacher in making decisions while enacting the curriculum. It is our view that, regardless of the authors' intentions, curriculum resources position the teacher in relation to the resources through the way they communicate with the teacher. The dimensions of our analytical framework are summarized in figure 1.

Author Intended Curriculum/lesson	Voice of the Text
Mathematical emphasis or focus Task cognitive demand Instructional representations Instructional approach	Guiding action or providing information Making developers design decisions visible Anticipating student thinking Mathematical explications Support for teacher decision making

Figure 1. **Dimensions of analytical framework.**

Methods

The study consists of document analysis of five elementary mathematics curriculum programs. The five programs analyzed are summarized in figure 2. A brief description of each follows.

Abb.	Curriculum Title (Edition)	Developers/Authors	Current publisher
EM	Everyday Mathematics (3rd Edition)	University of Chicago School Mathematics Project (P.I)	Wright Group / McGraw-Hill
INV	Investigations in Numbers, Data, and Space (2nd Edition)	TERC (Susan Jo Russell, P.I.)	Pearson
SF	Scott Foresman Mathematics	Scott Foresman/Pearson	Pearson
SM	Singapore Mathematics (Standards Edition)	Singapore Ministry of Education and Singapore Math.com	Marshall Cavendish International
TB	Trailblazers (3rd Edition)	TIMS at University of Illinois at Chicago	Kendall Hunt

Figure 2. The five programs analyzed.

Everyday Mathematics (EM)

Everyday Mathematics is packaged as a comprehensive curriculum for pre-kindergarten through sixth grade mathematics instruction. The first edition of *Everyday Mathematics* was developed by the University of Chicago School Mathematics Project from 1985 to 1988, and it is now in its third published edition. Its development was funded as one of NSF's Instructional Materials Development Projects (IMDP). The *Everyday Mathematics* curriculum emphasizes the development of mathematical themes or "habits of mind" throughout a student's elementary school experience. For this reason, lesson topics are often described as "spiraling"—that is, topics are visited briefly each year, to reinforce consistent meaning-making (rather than teaching each topic to mastery before moving onward to new material). *Everyday Mathematics* expects students to learn basic arithmetic facts and foundational algorithms, but the curriculum also expects students and teachers to understand contexts and meaning of facts and algorithms. Therefore, problem-solving is also explored with "real-world" motivations

Investigations in Numbers, Data, and Space (INV)

Investigations is a K-5 series that was developed by staff of the Education Research Center, TERC, and now in its second edition. The staff is composed of 2 co-principal investigators, 16 authors, 4 administrative staff, 11 contributing authors, 1 technology staff, 4 classroom field staff, and 52 collaborating teachers. The first edition was published between 1994 and 1998 with funding from NSF. *Investigations* has a modular organization with 9 units for each of grade 3-5. The concepts are not treated in the same order from one grade to another.

The *Investigations* package includes a teacher's guide, implementation guide, a resource binder, and student book. The teacher's guide contains information about what teachers and students ought to do to enact the various lessons. Each of the nine units for each grade is made up of 3-4 investigations and each investigation is made of 4-5 sessions (lessons). The beginning of each session contains Math Focus Points that outlines what students have to achieve at the end, a list of vocabulary (if available) to be introduced to students, the plan of the day which includes students' activities, allocated time, how it is to be managed (whether whole class, small groups, partner and individual work), and the materials needed for each of these activities. Each session consists of a combination of *ten-minute math, activity*, *discussion*, *math workshop*, *ongoing assessment*, *differentiation* and *session follow-up*. Also, the teacher's guide contains pictures of the student activity book, resource master, graphs, etc. on the left or right margins. It also contains professional development for teachers in the form of *teacher***/***teaching/math/algebra notes*, and *dialogue boxes* to provide: further information or rational why some routes are taken, insight about the mathematics students are to learn, students thinking about the task under investigation, and suggestions for how teachers might approach some difficult concepts to develop students understanding. This professional development part creates learning opportunities for teachers.

Scott Foresman (SF)

Scott Foresman, originally developed by the publishing company Scott Foresman, is now owned and marketed by Pearson. Each grade comes with four, large hardback spiral bound books. The entire year is comprised of 12 units, and each book contains 3 units. Students have a textbook and a workbook, there are transparencies and "Problem of the Day" cards included with the teachers' guide as well. Each book shows the entire "table on contents" for the year, a "lesson planner" for each chapter broken down to the lesson level. Each chapter is divided into sections A, B, and C and the lesson planner reflects this. The lesson planner also points the teacher to "Resources in the Student Book" and "Daily Real World Problem Solving" examples. Additionally, for each chapter there is an "Assessment, Intervention, Test Prep" section, identifying assessment resources, for diagnosing readiness, ongoing assessment, and formal evaluation and correlating each lesson to assessments, interventions, and standardized test skills. Additionally, each chapter has a "Skills Trace," which traces the chapter concept (i.e. division) before, during, and after this chapter. "Math Background and Teaching Tips" are also provided on the section level for each chapter.

Singapore Math (SM)

Singapore Math is the name commonly used to refer to Primary Mathematics, a series spanning from first to sixth grade and published by Marshall Cavendish International, a publishing company based in Singapore. The textbook and workbook match those that were written by a team assembled by the Ministry of Education (MOE) in Singapore and used in all Singapore schools until 2001. The teacher's guides were written in the US specifically for the North American audience. There are two versions of Singapore Math designed especially for use in the US and Canada. The Standards version is written to match the Mathematics Contents Standards for California Public Schools and features some additional content not contained in the curriculum used in Singapore. The US version matches the materials used in Singapore; the spellings are changed to match US conventions and a few sections were added to provide students practice in working with customary measurements. The distributor website suggests that schools should use the Standards Edition; for this reason we analyzed the California Standards here. The textbooks, workbooks, and teachers' guides are available separately on the website. The teachers' guides suggest assignments in both books. The textbook is designed to be used during class, and teachers are instructed to refer to examples in the textbook and assign tasks from this book during class. The homework assignments are contained in the workbook, which is more repetitive and generally features decontextualized problems.

The curriculum is integral, meaning that it is meant to be implemented in order. Each chapter begins with a chapter introduction, explaining the overarching ideas of the chapter and often referring to representations that students should have been exposed to in earlier grade. The written lesson is prefaced with objectives, alignment to the California Standards, a list of vocabulary words, and sometimes a note about the lesson or a list of materials. The lesson itself is presented in two columns, with written instructions for the teacher on the left, and illustrations and/or answers given on the left.

Trailblazers (TB)

Math Trailblazers (Trailblazers) is an elementary (K-5) mathematics curriculum developed by the Teaching Integrated Mathematics and Science Project (TIMS) at the University of Illinois at Chicago, with funding from NSF. Trailblazers is aligned with reform recommendations, especially those by the National Council of Teachers of Mathematics (known as NCTM Standards). Trailblazers also integrates mathematics with other disciplines, especially science and language arts.

Each grade has 16 – 20 units; each unit includes 5-9 lessons. The curriculum suggests that each lesson spend 1-5 sessions (mostly 1-2) to complete the activities. The curriculum has resources for students as well as those for teachers. Materials for students are Student Guide, Discovery Assignment Book (grades 3-5), and Adventure Book (a collection of stories on math and science concepts). Materials for teachers include Unit Resource Guides, Facts Resource Guide, Teacher Implementation Guide, and Teacher Resource CD.

Unit Resource Guides provide information and instructions for day-to-day teaching. Each Unit Resource Guide also includes general information about the unit, such as unit outline and pacing suggestions, background information about the main topics or mathematical ideas of the unit, assessment indicators, and daily practice problems of the unit. It even includes a letter to send to students' parents in order to let them know the focus and main ideas of the unit. Facts Resource Guide, compiling components of math facts program (Daily Practice Problems - DPP), explains the curriculum's philosophy and approaches to facts and skills and how to use DPP throughout the year. Teacher Implementation Guide is a comprehensive set of resources including philosophy of the curriculum, the overview of each unit, assessment, math facts and practice, and *TIMS tutors.* While each lesson guide includes explanations about the mathematics students learn, *TIMS tutors* provide in-depth explanations of the mathematical concepts and ideas behind the curriculum over more than 100 pages in each grade.

Coding

The first step in coding was to consider the model curriculum by looking across a set of lessons and determining the model lesson for each lesson analyzed. This analysis gave us, to the extent it could be interpreted, an image of the mathematical activity intended by the authors along with an understanding of the demands teachers might face in orchestrating these activities.

Cognitive demand. For each lesson, the coder identified the two main tasks of the lesson (those expected to take the majority of time) and noted the cognitive demand of each, following the descriptors provided in Stein, et al. (2000). Tasks were coded as Doing Mathematics (DM), Procedures with Connections (PWC), Procedures without Connections (PWOC) or Memorization (M).

Instructional Representations. The primary instructional representations or models used during the lesson were also described and indexed. These have not yet been analyzed and are not discussed in this paper.

Instructional Approach. The instructional approach was determined for each lesson by identifying the roles teachers and students are expected to play during the lesson and by inferring, through analysis of these roles, how student learning is expected to take place.

Mathematical Emphasis. To date, the mathematical emphasis of the curriculum has not been fully analyzed for all five programs and is not discussed in this paper. Analysis will include examination of the learning goals for Numbers, Operations, and Algebra, as stated by the authors and as enacted through the components of the lessons, consideration of the cognitive demand in the curriculum, and assessment of what mathematical knowledge is valued.

The second phase of coding involved close examination of the teacher's guide to uncover the voice of the text. Specifically, we look at the way the guide communicated with the teacher, what types of guidance it provided, and what the guidance and its placement in the lessons suggest about the underlying assumptions about teaching and curriculum use. We were particularly interested in patterns in how different kinds of messages to teachers were proportioned within the lessons, which led us to counting and coding sentences and phrases in the teacher's guide.

After establishing guidelines for counting relevant sentences and phrases in the teacher's guide, we coded each sentence/phrase as belonging to one of the first four categories described in figure 3. We allowed for double coding of sentences/phrases when both messages appeared to be equally emphasized. The Decision Making code (D) was applied to appropriate sentences along with one of the first four codes.

Code and Explanation	Inclusion Criteria
Directing Action	Statements intended a) direct teacher and student action, b) provide information
Explain Rationale The intent of the material in this category is to help teachers understand the purpose behind the lesson and make sound decisions while teaching the lesson that are inline with the intent of the curriculum designers.	Statements intended to help the teacher understand design decisions in the curriculum by a) providing transparency about or rationale for design decisions, b) explaining the purpose or intent of an activity/task, or c) indicating how specific topic/content connects to previously taught or future content
Student Thinking The intent of the material in this category is to focus the teacher's attention on student thinking and understanding and, in some cases, guide the teacher to respond to student thinking.	Statements aimed at focusing attention on student thinking and/or understanding by a) specifying what students need to understand or be able to do, b) indicating ways students might respond to a task, including difficulties they are likely to encounter, solution strategies they might attempt, or likely responses they might provide, c) indicating desired elements of student thinking the teacher should look for, or d) assisting the teacher in interpreting students' responses and in responding to students.
Explain Math The intent of the material in this category is often to help the teacher understand the important mathematical ideas and, in some cases, identify the central mathematical ideas.	Statements intended to communicate mathematical concepts, relationships, or insights to the teacher or to specify the mathematical importance of something.
Decision Making The intent of the material in this category is to communicate to the teacher that flexibility in how the program is used is expected or that they are expected to make decisions based on their assessment of their students' needs.	Any material given one of the four previous codes, could also be assigned a "decision making" code (D). Statements coded in this way suggest the need for teacher decision making. This category includes: a) indications that the teacher should make a decision about how or when to uses some aspect of the curriculum, or b) some sort of guidance on how to make the decision

Figure 3. **Explanations of codes used for voice of the text.**

Inter-Rater Checks

Initially, each program was analyzed and coded by two members of the research team; coding decisions were then discussed by the entire team. During the coding discussions, coding guidelines were revised and clarified, to guide future coding. Midway through the process, each team member coded one lesson from each program individually. These codes were compared during a team meeting and, when disagreements occurred, differences were discussed and resolved and new guidelines were articulated. During these discussions, we made every effort to consider the application of the evolving coding rules to each program. For instance, INV provides a statement about how long each portion of the lesson should be allotted. SF provides an image of a clock face shaded to show the amount of time required. As a result, we counted both of these references to time (one in sentence form and one in image form) as once phrase, coding both as directing action.

Analysis

Based on the analysis, each researcher analyzed the data for a single program in order to characterize the model lesson and curriculum. Using a spreadsheet, the assigned codes for each lesson were combined and total number of sentences/phrases per lesson was calculated. Similarly, the percentage of sentences devoted to each type of statement per lesson was also calculated. The data for all five programs were then combined to look comparatively at the curricula.

Findings

In this paper, we present several findings from our cross-curricular analysis of the five programs. These patterns are preliminary at this point because they result from close examination of only 9 lessons, three from each grade 3-5. Although we found substantial internal consistency in how the lessons were structured and the voice of the text, we believe that examining a larger sample would ensure the accuracy of our assessment of cognitive demand and instructional approach.

Modeling the Author-Intended Curriculum

In order to describe the model curriculum, we examined the mathematical emphasis, cognitive demand of the tasks in the lessons analyzed, and the primary instructional approaches used comparatively. Taken together, these elements reveal the mathematical and pedagogical complexity of the curriculum and the role the teacher is expected to play in fostering this level of instruction. We begin with the cognitive demand of the tasks in the 9 lessons analyzed. Table 1 gives the total number of tasks analyzed for each program and shows how they were distributed across the four cognitive demand categories.

Our findings around this analysis involve two irregularities. First, as the footnote suggests, classifying the tasks in SF was challenging. First, each lesson focused on a single mathematical skill that was generally introduced in a way that made conceptual connections and then practiced in a way that placed primary emphasis on the procedures. We refer to these tasks as procedures with superficial connections because, when compared to the PCW tasks in the other curricula, the connections were substantially less robust. Second, some of the tasks in TB were difficult to classify because they had elements of DM and PCW. We referred to these tasks as blended PWC/DM.

Table 1

Cognitive Demand of Primary Tasks in Each Curriculum

n per curriculum	Memorization	PWOC	PWC	Doing Math
EM n=18	4 (22%)	4 (22%)	9 (50%)	1 (6%)
INV n=15	-	-	5 (45%)	6 (55%)
SF n=18	-	9 (50%)	9 (50%)*	-
SM n=21	-	10 (48%)	9 (43%)	2 (9%)
TB n=15	-	-	11 (73%) 2 (13%) PWC/DM	2 (13%)

*All of the SF introductory tasks could be characterized as procedures with *superficial* connections. The main task introduces a concept or skill making connections to underlying meaning, visual models, or suggests multiple solution paths; the lesson proceeds to practice of skill in a routinized, decontextualized way We categorized these as 50% PWOC and 50% PWC.

For EM and TB, PWC is the dominant choice by significant amounts. The analyzed tasks in SF and SM are more equally divided between PWOC and PWC. Two of the programs, INV and TB, contain only high-level tasks. EM was the only program with tasks that spanned all four categories.

In their analysis of INV and EM, Stein and Kim (2009) argue that high demand tasks, especially tasks classified as *doing mathematics*, place greater demand on the capacity of the teacher to manage them. Our analysis of the teacher's role as described in the curriculum guide, corresponds with the analysis of the cognitive demand. Figure 4 locates the five programs according to the distribution of cognitive demand category, along the vertical access, and according to the predominant role of the teacher, along the horizontal access. These role classifications were determined by the guidance provided in the teachers' guide.

Telling, showing, directing is the predominant role for the teacher in SM and SF. Both programs position the teacher as the provider of information and rules to follow. The corresponding student's role is to follow the teacher's guidance. The *guiding* role, which is the predominant mode in EM, gives the teacher a significantly less didactic role than telling, but still, the teacher is framed as the primary shaper of classroom interactions. The teacher's guide gives the teacher questions to ask designed to prompt student thinking along with answers to expect. The tasks that dominate the lessons in TB and INV position the teacher in a much less central way that the previous two roles. In both these programs, students are expected to explore, make observations, develop their own approaches to solving problems. We label the predominant teacher role in TB as *facilitating* because it describes the way the teacher is guided to foster student exploration and meaning making. The teacher's guide provides suggestions for the teacher on ways to encourage and facilitate students' exploratory work. We refer to the primary teacher's role in INV as *orchestrating* to indicate the appearance of a role that is even less participatory and visibly active than facilitating. Our analysis suggests that the authors intended student talk and students' understandings of mathematical ideas to dominate interactions in the INV classroom. The teacher's role is critical in fostering and managing the discourse, but much of it happens from the sidelines.

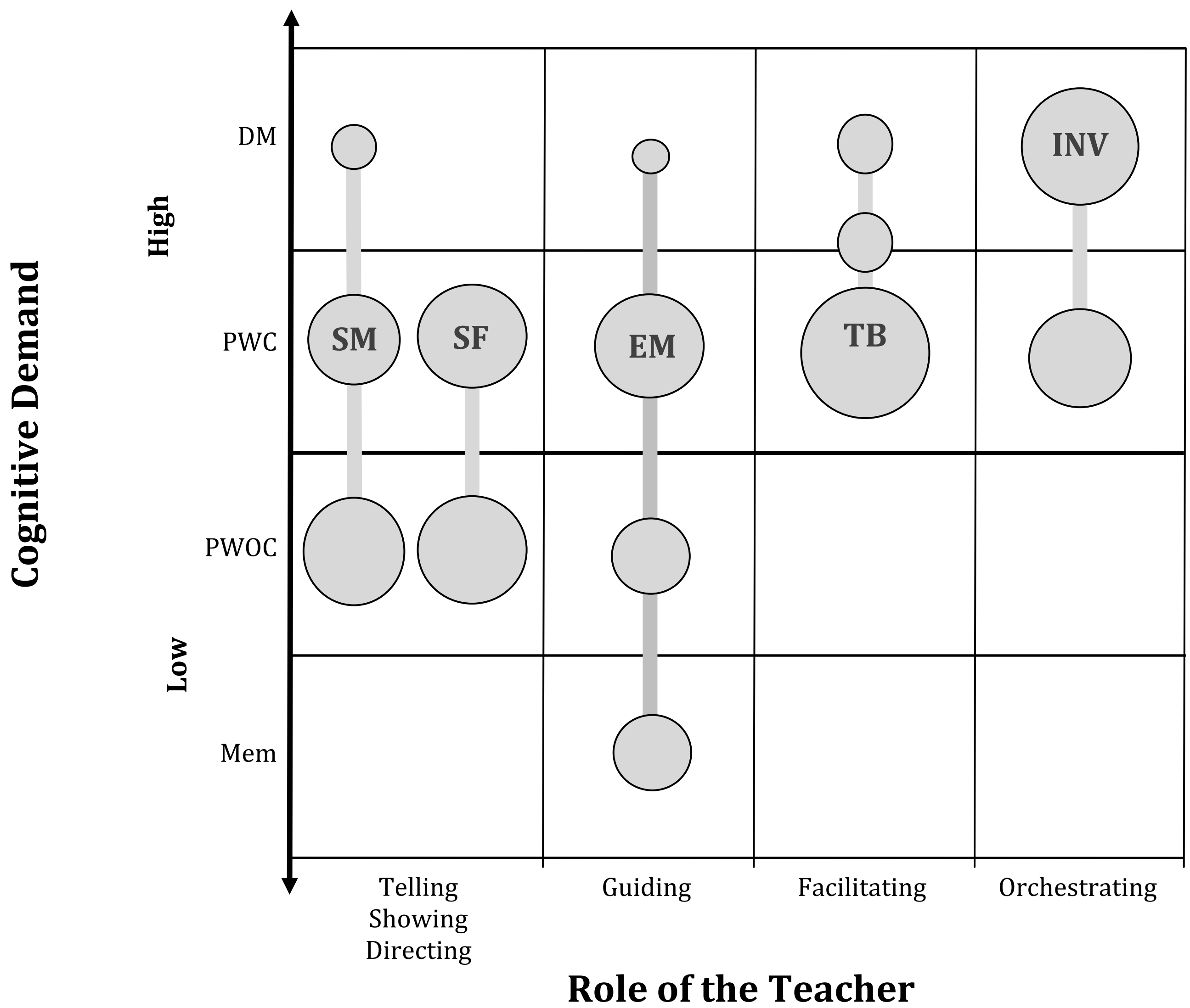

Figure 4. **The relationship between cognitive demand and the role of the teacher.**

For the most part, the relationship between the level of cognitive demand and the role of the teacher is not surprising; the programs with patterns of high demand tasks also call for more demanding pedagogy. This combined representation of pedagogy and task helps to illustrate an important challenge for curriculum implementation. As the cognitive demand is increased, students are expected to become more intellectually active in the tasks. The discourse becomes less predictable and teachers must make pedagogical decisions on the fly. Managing high demand tasks requires greater teacher capacity (Stein & Kim, 2009). Figure 4 illustrates an additional challenge that comes with high-demand teacher roles that accompany the increase in cognitive demand—as the task becomes more difficult to predict and manage, the teacher is expected to adopt a less directive and more facilitative role.

SM and SF do not fully fit this pattern. Although all if the analyzed tasks were coded as PWOC and above, the teacher's role is primarily one of showing, telling, and directing. The directive role assigned to the teacher is likely to have one of two effects. It could subtly invite the teacher to reduce the cognitive demand of the tasks when enacting them. Or it could support the teacher in maintaining the level of cognitive demand. We return to this question later when discussing the supports provided by the curriculum.

Finally, we turn to a comparison of how the students' roles are conceptualized in these five programs. Figure 5 lists the major activities students are expected to engage in during lessons. The large number and breadth of activities expected of students in TB, INV, and EM classrooms stands out in this figure when compared to the narrow range in SF and SM. We conjecture that this variety places additional demands on the teacher, requiring focus on multiple areas of skill development.

Primary Student Roles	EM	INV	SF	SM	TB
Follow Teacher math instructions	f		P	P	
Independent practice	P		P	P	P
Solve routine problems	P	P	P	P	P
Solve nonroutine problems	f	P			P
Select solution strategy	f	P			P
Invent solution strategies/approach		P			f
Consider/compare differing strategies	f	P	f		P
Observe/find patterns	f	f			P
Make generalizations					f
Explain solutions to others	f	P			P
Collaborate with others	P	P	f		P
Interpret and use visual models	f	P	f	f	P

Primary student roles (indicated with P) refers to activities students are asked to do this at least once during any lesson. Frequent student roles (indicated with f) refers to activities students are asked to do often, but not necessarily every lesson.

Figure 5. **Primary student roles found in each program.**

Supports in the Curriculum

In this section, we present initial findings from our analysis of how the different teacher guides communicate with the teacher. The instructional pages of 9 lessons for each program were coded at the sentence/phrase level to determine the proportion of guidance devoted to each of the following types of guidance: directing action (or providing information), explaining the rationale for design decisions, anticipating student thinking, explaining the mathematics, and supporting teacher decision making. For each lesson, the proportion of sentence/phrases devoted to each type was calculated in percents. Table 2 provides the mean, median, and range percentage for each guidance type by curriculum program.

Several patterns stand out in table 2. First, based on the number of sentences and phrases, it is evident that SM and SF provide the teacher with minimal guidance. The lessons in SF contained an average of 83.3 sentences; whereas, the lessons in SM contained only 59.8 sentences. Further, the type of guidance these two programs provided the teacher was predominantly directing action and providing information (over 85% in each program). It is worth noting (and will be discussed in later analysis) that these two programs approach directing the teacher's actions in very different ways. The limited amount of teacher support provided by the curriculum corresponds with the lack of pedagogical sophistication required to enact both of the se programs, when compared to the others.

On the other end of the spectrum, EM, TB and INV seem to provide substantially more guidance than SM and SF and general and more guidance designed to be educative for the teacher and, subsequently, support the teacher when enacting the lesson (Ball & Cohen, 1996). The lessons in INV included an average of 114.8 and in EM 116.4 sentences. Approximately 75% of the guidance provided in these two programs was devoted to directing action. These two programs seem to be fairly comparable in their attention to other four types of guidance, although INV stands out as placing the most emphasis on anticipating student thinking.

Our findings with respect to INV surprised us, given our knowledge of the program and Stein & Kim's (2009) analysis. For this reason, the codes assigned to these programs were confirmed by multiple team members. Our best explanation of this finding is the fact that the format of the INV teacher's guide was changed substantially for the second edition. Many of the teacher notes that were previously integrated into the lesson now appear in a separate section of the guide entitled *Professional Development.* The teacher notes under the heading *Math Notes* still appear on the lesson pages. Because INV seems to require the most pedagogical sophistication to enact, the reduction in available supports in the second edition might have implications for use.

From this analysis, TB appears to stand out in a number of ways. On the far end of the spectrum, the lessons in TB included an average of 128.6 sentences and only 65% focused on directing teacher action. As the table indicates, the TB teacher's guide attends to all other categories of teacher guidance in substantial ways.

It is important to emphasize that even when two or more provide comparable proportions of guidance in a particular category, the approach can vary significantly. We believe that where this type of information is located in relation to other guidance and how it is presented may matter for how teachers use it.

Implications for Teacher Use

The variation in the type of teacher support found in these five programs is striking. The next logical question is how do teachers use these resources and does the presence of educative supports make a difference? Moreover, what kind of teacher capacity or contextual support might enhance use of these resources? Stein and Kaufman (2010) found that, in the case of some programs, use of the educative guidance in the teacher's guide can relate to higher quality implementation. Our analysis of these five programs provides us with a base for examining on how teachers use the different supports in their teacher's guides and how their use affects their classroom interactions. These questions will be the focus of the next phase of our study.

Table 2

Variations in Types of Guidance for Teachers across Five Curriculum Programs

			Percent of Total Number of Sentences/Phrases Devoted to. . .				
	Sentences/ Phrases per Lesson		**Directing Action**	**Explaining Rationale**	**Anticipating Student thinking**	**Explaining Math**	**Supporting Decision Making**
Everyday Math	116.4	Mean	78.6	8.3	7.5	5.6	7.5
		Median	77.4	8.0	7.9	4.3	7.8
		Range	68.2-87.6	5.4-13.9	0-12.9	0.0-18.9	3.4-11.9
Investigations	114.8	Mean	74.3	6.8	12.8	3.9	2.2
		Median	76.8	5.4	10.9	1.5	1.5
		Range	61.8-81.4	1.5-12.3	7.8-23.3	0.0-10.8	0.0-5.5
Scott Foresman	83.3	Mean	86.5	0.5	10.0	3.0	2.2
		Median	86.6	0.0	9.8	2.5	1.4
		Range	81.4-92.0	0.0-2.2	5.3-16.9	1.2-7.8	1.2-4.6
Singapore Math	59.8	Mean	87.91	1.0	5.2	5.9	0.8
		Median	89.7	0.0	6.5	4.7	0.0
		Range	76.6-95.3	0.0-3.2	0.0-9.7	1.3-13.3	0.0-2.6
Math Trailblazers	128.6	Mean	65.5	14.0	13.2	10.5	5.8
		Median	63.5	12.2	14.1	10.8	5.5
		Range	51.1-86.5	8.1-23.1	2.7-25.9	0.0-26.3	1.2-13.1

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